Booster, AGS, and RHIC Parameters "as Run" in FY2000

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The Tables in this note contain the nominal "as run" parameter values for the FY2000 RHIC Run.

1 Basic Formulae

1.1 Mass, Energy, Momentum, Rigidity, and Frequency

A Gold ion with charge eQ has $N = 197$ Nucleons, $Z = 79$ Protons, and $(Z - Q)$ electrons. (Here Q is an integer and e is the charge of a single proton.) The mass and energy are

$$m = au - Qm_e + E_b/c^2, \quad E = \sqrt{p^2c^2 + m^2c^4} \tag{1}$$

where $a = 196.966552$ is the atomic mass [1, 2] of the neutral Gold atom, $u = 931.494013$ MeV$/c^2$ is the unified atomic mass unit [3], $m_ec^2 = .510998902$ MeV is the electron mass [3], and p is the momentum. E_b is the binding energy of the Q electrons removed from the neutral Gold atom. This amounts to 0.327 MeV for the fully stripped Gold ion as calculated by Trbojevic [4]. The kinetic energy is

$$W = E - mc^2. \tag{2}$$

In terms of W, the momentum and energy are

$$cp = \sqrt{W^2 + 2mc^2W}, \quad E = mc^2 + W. \tag{3}$$

The magnetic rigidity of the ion in units of Tm is

$$B\rho = kp/Q \tag{4}$$

where $k = 10^9/299792458$ and p is the momentum in units of GeV/c. The relativistic parameters β and γ, and the revolution frequency of the ion are

$$\beta = cp/E, \quad \gamma = E/(mc^2), \quad f = c\beta/(2\pi R) \tag{5}$$

where R is the machine radius. The angular frequency is $\omega = 2\pi f$. We also define the phase-slip factor

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \tag{6}$$

where γ_t is the transition gamma.

1.2 RF Parameters

Various RF Bucket and Bunch parameters are as follows. The half-height of the bucket is

$$\Delta E = \left(\frac{h\omega_s}{8}\right) A_S \left| \frac{(\pi - 2\phi_s)\sin\phi_s - 2\cos\phi_s}{2} \right|^{1/2} \tag{7}$$

where the subscripts "s" indicate parameter values for the synchronous particle, h is the RF harmonic number, ϕ_s is the synchronous phase, and

$$A_S = 8\,\frac{R_s}{hc}\left\{\frac{2eQV_gE_s}{\pi h|\eta_s|}\right\}^{1/2}. \tag{8}$$

Here V_g is the total RF gap voltage per turn and A_S is the area of the corresponding Stationary bucket. The synchronous phase is given by

$$V_g \sin\phi_s = 2\pi R_s \rho_s \dot{B}/c \tag{9}$$

where ρ_s is the radius of curvature, B is the magnetic field and $\dot{B} = dB/dt$. Employing Gaussian units (R_s and ρ_s in cm, $c = 2.99792458 \times 10^{10}$ cm/s, and $\dot{B}$ in G/s) gives $V_g \sin\phi_s$ in Statvolts. Multiplying by 299.792458 then gives $V_g \sin\phi_s$ in Volts.

The width of the bucket is

$$\Delta t = \frac{|\pi - \phi_s - \phi_e|}{h\omega_s} \tag{10}$$

where the phase ϕ_e satisfies

$$\cos\phi_e - \cos(\pi - \phi_s) = -\{\phi_e - (\pi - \phi_s)\}\sin\phi_s. \tag{11}$$

The half-height and full width of the bunch matched to the bucket are given by

$$\Delta E_m = \left(\frac{h\omega_s}{8}\right) A_S \left|\frac{\cos\phi_m - \cos\phi_s + (\phi_m - \phi_s)\sin\phi_s}{2}\right|^{1/2} \tag{12}$$

and

$$\Delta t = \frac{|\phi_m - \phi_e|}{h\omega_s} \tag{13}$$

where the phase ϕ_e satisfies

$$\cos\phi_m - \cos\phi_e + (\phi_m - \phi_e)\sin\phi_s = 0. \tag{14}$$

For a bunch matched to a stationary bucket the half-height and width are given by

$$\Delta E_m = \left(\frac{h\omega_s}{8}\right) A_S \left|\frac{\cos\phi_m \mp 1}{2}\right|^{1/2}, \quad \Delta t = \frac{|2\phi_m|}{h\omega_s} \tag{15}$$

where the "$-$" and "$+$" signs are for buckets below and above transition respectively. The area of a small bunch in a stationary bucket is approximately

$$A_b = \left(\frac{\pi A_S}{16}\right)\phi_m^2. \tag{16}$$

2 Lattice Parameters

Parameter	Booster	AGS	RHIC	Unit
Radius R	128.4580/4	128.4580	3833.845/(2π)	m
ρ	13.8656	85.378	242.7806	m
$\gamma_{\rm tr}$	4.806	8.5	22.89	
Q_H, Q_V	4.757, 4.777	8.78, 8.72	28.19, 29.18	
Max β_H, β_V	13.5, 13.2	22.3, 22.2	48.6, 47.4	m
R/Q_H, R/Q_V	6.75, 6.72	14.6, 14.7	21.6, 20.9	m
Max D_H	2.90	2.17	1.81	m
$R/\gamma_{\rm tr}^2$	1.39	1.78	1.16	m

Here the machine radii R are based on the nominal circumference, 3833.845 m, of RHIC. If we let R_b, R_a, R_r be the radii of Booster, AGS, and RHIC respectively, then

$$R_a = 4R_r/19, \quad R_b = R_a/4. \tag{17}$$

We note that this gives an AGS radius approximately 5 mm larger than that reported by Bleser [5]. The other Booster and AGS parameters were obtained from MAD runs. The RHIC parameters are taken from Ref. [6].

3 Gold Parameters in Booster, AGS, and RHIC

The "as run" parameters values in the following tables are calculated assuming that:

1. The kinetic energy of Gold ions (Au^{32+}) at Booster injection is $W = 182.13$ MeV.
2. The magnetic rigidity at booster extraction is $B\rho = 9.152950$ Tm.
3. The energy lost by a gold ion in the stripping foil between the Booster and AGS is $\delta E = 4.1$ MeV per nucleon. This number is calculated in Ref. [7].
4. The value of Gamma at AGS extraction and RHIC injection is $\gamma = 10.264620$. This was calculated from measurements by Ahrens and Smith [8] of frequency and radial offset in the RHIC rings. (Note that the nominal value for the FY2000 run was supposed to be $\gamma = 0.85 \times 12.09 = 10.2765$).
5. The value of Gamma at RHIC Store is $\gamma = 70$.

The Bunch and Bucket parameters were obtained from the Computer Program "bbat".

In the following tables, more digits are given for some parameters than would be warranted by the measurement precision; this is done for computational convenience. The notation "$/N$" in the Units column means "per nucleon".

3.1 Gold in Booster

Parameter	Injection	Extraction	Unit
Charge Q	32	32	
Mass m	183.456812	183.456812	GeV/c^2
Kinetic Energy W	0.18213/197	0.1011721	GeV/N
Momentum p	0.04150632	0.4457235	GeV/c/N
Energy E	0.9321774	1.0324250	GeV/N
Rigidity $B\rho$	0.852334	9.152950	Tm
β	0.04452620	0.43172485	
γ	1.0009928	1.108641	
η	-0.955	-0.770	
Norm ϵ_H (95%)	8.2π	8.2π	mm mrad
Norm ϵ_V (95%)	3.9π	3.9π	mm mrad
Rf harmonic h	6	6	
Rf	0.396923	3.848558	MHz
No. of Bunches	6	6	
Bunch Spacing	2519.380	259.838	ns
Ions/bunch	1.23/6	1.00/6	10^9
Single Bunch Area	0.04/6	0.04/6	eV s/N
Bunch Width	1400	45	ns
Bunch Half-Height	0.62	19	MeV
Bucket Half-Height	0.80	51	MeV
Gap Volts	0.5	30	KV
Bucket Area	0.079/6	0.350/6	eV s/N

The normalized transverse emittances listed for the Booster follow from the assumption that during multi-turn injection the horizontal and vertical apertures are completely filled. The horizontal and vertical acceptances (un-normalized) of the Booster are 185π and 87π mm mrad respectively.

The Bunch Area in Booster is based on the measured Bunch Width at extraction with $\dot{B} = 37$ G/ms and $V_g = 30$ kV.

Note that $\dot{B}$ is zero at injection in Booster. In past runs, $\dot{B}$ was typically 1–2 G/ms at injection; for the FY2000 run, beam was injected onto a 6 ms porch with $\dot{B} = 0$.

3.2 Gold in AGS

Parameter	Injection	Transition	Extraction	Unit
Charge Q	77	77	77	
Mass m	183.434144	183.434144	183.434144	GeV/c^2
Kinetic Energy W	0.0970596	6.983533	8.626638	GeV/N
Momentum p	0.4360875	7.859708	9.512311	GeV/c/N
Energy E	1.0281974	7.914671	9.557776	GeV/N
Rigidity $B\rho$	3.721589	67.075078	81.178464	Tm
β	0.42412821	0.99305547	0.99524316	
γ	1.104238	8.5000	10.264620	
η	-0.806	0.0	0.00435	
Norm ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
Norm ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
Rf harmonic h	24	12	12	
Rf	3.780839	4.426235	4.435986	MHz
No. of Bunches	24	4	4	
Bunch Spacing	264.492	677.777	676.287	ns
Ions/bunch	0.52/6	0.50	0.50	10^9
Single Bunch Area	0.172/6		0.3	eV s/N
Bunch Width	60		17	ns
Bunch Half-Height	60		2210	MeV
Bucket Half-Height	172		18700	MeV
Gap Volts	320		200	KV
Bucket Area	1.76/6		27.3	eV s/N

The increase in longitudinal emittance between Booster and AGS is due to the increase in energy spread upon passing through the stripping foil between the two machines. The increase in longitudinal emittance between AGS injection and extraction is due to the bunch spacing mismatch caused by the energy loss in the stripping foil.

$\dot{B}$ is zero at injection and extraction in AGS.

3.3 Gold in RHIC

Parameter	Injection	Transition	Store	Unit
Charge Q	79	79	79	
Mass m	183.433122	183.433122	183.433122	GeV/c^2
Kinetic Energy W	8.626590	20.382493	64.248149	GeV/N
Momentum p	9.512258	21.293276	65.172631	GeV/c/N
Energy E	9.557722	21.313625	65.179282	GeV/N
Rigidity $B\rho$	79.122872	177.117274	542.105338	Tm
β	0.99524316	0.99904526	0.99989795	
γ	10.264620	22.8900	70.000000	
η	-0.00758	0.0	0.00170	
Norm ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
Norm ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
Rf harmonic h	360	360	360	
Rf	28.016756	28.123788	28.147792	MHz
No. of Bunches	60	60	60	
Bunch Spacing	214.158	213.343	213.161	ns
Ions/bunch	0.50	0.50	0.50	10^9
Single Bunch Area	0.3		0.7	eV s/N
Bunch Width	17		11	ns
Bunch Half-Height	2230		8170	MeV
Bucket Half-Height	3210		17800	MeV
Gap Volts	300		300	KV
Bucket Area	0.741		4.1	eV s/N

Note that the harmonic 2520 RF system was not operational for the FY2000 run, so there was no "re-bucketing" of the stored beam.

References

[1] David R. Lide (Editor-in-Chief), Handbook of Chemistry and Physics, 80th Edition, 1999–2000, CRC Press LLC, 1999, pp. 1-10 through 1-12.

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[3] D.E. Groom, et al., The European Physical Journal C 15, 73 (2000)

[4] Dejan Trbojevic, "Rest Mass of the Fully Stripped Gold Ions", RHIC/AP/20, January 13, 1994.

[5] E.J. Bleser, "Where are the AGS Magnets", Accelerator Division Technical Note 215, May 20, 1985.

[6] W. Fischer and S. Peggs, "RHIC Parameters", Revision of March 18, 1997.

[7] C.J. Gardner, "Heavy Ion Parameters for 1997–98, and Some Preliminary Parameters for BAF", AGS/AD/Tech. Note No. 472, November 6, 1997.

[8] L.A. Ahrens and K.S. Smith, Logbook FE.301.2.0002, pp. 41–43, August 31, 2000. The measured frequency at injection in the Blue and Yellow Rings was $hf = 28.016745(15)$ MHz ($h = 360$) with a measured radial offset of 0.25 mm. At the nominal RHIC radius this gives $hf = 28.016756$ MHz. The corresponding values of γ and $B\rho$ are 10.264620 and 79.122872 Tm.